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WAVES IN AN INHOMOGENEOUS FLUID IN THE PRESENCE OF A DOCK

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We investigate the propagation of waves generated by oscillations of a section of the bottom of a tank through a two-layer fluid, in the presence of a dock. Wave motions in an inhomogeneous fluid generated by displacement of a section of the bottom of a tank were studied in [1] where the upper surface of the fluid was assumed either to be completely free, or completely covered with ice. In the present paper we use the method given in [2] to investigate a similar problem under the assumption that the fluid surface is partly covered with an immovable rigid plate. The expressions obtained for the velocity potential are used to determine the form of the free surface and of the interface. We show that when the fluid is inhomogeneous, the wave amplitude on the free surface increases, while the presence of a plate reduces the amplitude of the surface waves, as well as of the internal waves in the region between the plate and the oscillating section of the bottom.

An immovable rigid plate occupying the region $y = h,\ x \leqslant -l,\ -\infty < z < \infty$ is situated at the surface of two-layer fluid in which the density and depth of the upper and lower layer are denoted, respectively, by ρ, h and ρ_1, H. The coordinate origin is situated at the interface and the y-axis is directed vertically upwards. The bottom section $y = -H,\ 0 \leqslant x \leqslant a,\ -\infty < z < \infty$ is deformed according to the law

$$y = \varepsilon \sin \frac{\pi}{a} x \operatorname{Re} [\exp i (kz - \omega t)]$$

where ε denotes the maximum deviation of the points lying on the median line of the oscillating section of the bottom and is assumed small compared with the depth $H + h$ of the fluid. The velocity potentials of the upper and lower fluid $F(x, y, z, t)$ and $F_1(x, y, z, t)$ must satisfy the following equations:

$$\Delta F(x, y, z, t) = 0, \quad h \geqslant y \geqslant 0; \quad \Delta F_1(x, y, z, t) = 0, \quad 0 \geqslant y \geqslant -H$$
$$(-\infty < x < \infty, \quad -\infty < z < \infty)$$

with the boundary conditions (g is acceleration due to gravity)

$$\frac{\partial^2 F}{\partial t^2} + g \frac{\partial F}{\partial y} = 0, \quad y = h, \quad x > -l, \quad -\infty < z < \infty$$

$$\partial F / \partial y = 0, \quad y = h, \quad x \leqslant -l, \quad -\infty < z < \infty$$

$$\frac{\partial F}{\partial y} = \frac{\partial F_1}{\partial y}, \quad \rho \frac{\partial^2 F}{\partial t^2} - g(\rho_1 - \rho) \frac{\partial F}{\partial y} = \rho_1 \frac{\partial^2 F_1}{\partial t^2}, \quad y = 0$$
$$-\infty < x < \infty, \quad -\infty < z < \infty$$

$$\frac{\partial F_1}{\partial y} = \begin{cases} 0, & -\infty < x < 0, \quad a < x < \infty \\ -\omega\varepsilon \sin \dfrac{\pi}{a} x \operatorname{Re} [i \exp i (kz - \omega t)], & 0 \leqslant x \leqslant a \end{cases}$$
$$y = -H, \quad -\infty < z < \infty$$

The motion of the fluid must be restricted near the point $(-l, h)$ lying away from the plate, and decay during its motion under the plate, i.e. $\partial F / \partial t$ in particular must be bounded at the edge of the plate.

We now write two boundary value problems for the functions $F(x, y, z, t)$ and $F_1(x, y, z, t)$, solve the first problem using the method given in [2], and the second problem with the help of Fourier transformation in x. The kernel $K(\alpha)$ of the resulting functional equation has the form

$$K(\alpha) = E(\gamma) / L(\gamma), \quad \gamma^2 = \alpha^2 + k^2$$
$$E(\gamma) = \gamma [(\gamma \operatorname{sh} \gamma h - \chi \operatorname{ch} \gamma h) \operatorname{sh} \gamma H - \chi_1 \operatorname{sh} \gamma h \operatorname{ch} \gamma H]$$
$$L(\gamma) = [(\gamma^2 + \beta\chi) \operatorname{sh} \gamma h - \gamma (\beta + \chi) \operatorname{ch} \gamma h] \operatorname{sh} \gamma H -$$
$$\chi_1 (\gamma \operatorname{sh} \gamma h - \beta \operatorname{ch} \gamma h) \operatorname{ch} \gamma H,$$
$$\beta = \frac{\omega^2}{g}, \quad \chi = \frac{\rho\beta}{\rho_1 - \rho}, \quad \chi_1 = \frac{\rho_1\beta}{\rho_1 - \rho}$$

When $\rho_1 \to \rho$, the expressions obtained for $F(x, y, z, t)$ and $F_1(x, y, z, t)$ coincide with the corresponding formulas of [2]; when $l \to \infty$ they become identical with the results obtained in [1].

The form of the free surface $\zeta(x, z, t)$ and the interface $\zeta_1(x, z, t)$ is obtained from the formulas [1]

$$\zeta(x, z, t) = -\beta\omega^{-2} \frac{\partial F}{\partial t}, \quad y = h$$

$$\zeta_1(x, z, t) = -\omega^{-2} \frac{\partial}{\partial t} (\chi_1 F_1 - \chi F), \quad y = 0$$

In the case when the distance between the edge of the plate and the oscillating section of the bottom exceeds the depth of the fluid, the perturbations of the free surface generated by the deformations of the bottom are written in the form

$$\zeta(x,z,t) = 2\varepsilon\beta\frac{\pi}{a}\Big\{f_1(\alpha_*,\gamma_*,h)\,[\sin\alpha_* x + \sin\alpha_*(x-a)] + f_1(\alpha_0,\gamma_0,h)\,[\sin\alpha_0 x + \sin\alpha_0(x-a)] + \frac{1}{2}\sum_{n=1}^{\infty}\frac{1+e^{\alpha_n a}}{A_n D_n} f_0(i\gamma_n,h)\,e^{-\alpha_n x}\Big\}\cos(kz-\omega t)$$

when $a < x < \infty$, in the form

$$\zeta(x,z,t) = 2\varepsilon\beta\frac{\pi}{a}\Big\{\frac{af_0(b,h)}{2\pi L(b)}\sin\frac{\pi}{a}x + f_1(\alpha_*,\gamma_*,h)\sin\alpha_* x + f_1(\alpha_0,\gamma_0,h)\times \sin\alpha_0 x + \frac{1}{2}\sum_{n=1}^{\infty}\frac{f_0(i\gamma_n,h)}{A_n D_n}\,[e^{\alpha_n(x-a)} + e^{-\alpha_n x}]\Big\}\cos(kz-\omega t)$$

when $0 \leqslant x \leqslant a$ and in the form

$$\zeta(x,z,t) = \varepsilon\beta\frac{\pi}{a}\sum_{n=1}^{\infty}\frac{1+e^{-\alpha_n a}}{A_n D_n} f_0(i\gamma_n,h)\,e^{\alpha_n x}\cos(kz-\omega t)$$

$$f_1(\alpha,\gamma,y) = \frac{f_0(\gamma,y)}{\alpha\delta(\gamma)}\left[\alpha^2-\frac{\pi^2}{a^2}\right]^{-1},\quad \delta(\gamma) = \frac{dL(\gamma)}{d\alpha}\alpha^{-1}$$

$$b = \left(k^2+\frac{\pi^2}{a^2}\right)^{1/2},\quad f_0(\gamma,y) = \frac{\chi_1}{\gamma}[\gamma\,\mathrm{ch}\,\gamma(h-y) - \beta\,\mathrm{sh}\,\gamma(h-y)]$$

$$\alpha_* = (\gamma_*^2-k^2)^{1/2},\quad \alpha_0 = (\gamma_0^2-k^2)^{1/2},\quad \alpha_n = (\gamma_n^2+k^2)^{1/2}$$

$$A_n = \alpha_n\delta(i\gamma_n),\quad D_n = \alpha_n^2+\frac{\pi^2}{a^2}$$

when $-l<x<0$. Here $\pm\gamma_*, \pm\gamma_0$ and $\pm i\gamma_n$ are the roots of the equation $L(\gamma)=0$).

The expressions for the perturbations of the interface can, in this case, be obtained from the formulas defining the perturbations of the free surface in the corresponding regions, by replacing $\beta f_1(\alpha,\gamma,h)$ and $\beta f_0(\gamma,h)$ by $\chi_1 f_2(\alpha,\gamma,0) - \chi f_1(\alpha,\gamma,0)$ and $\chi_1 R_0(\gamma,0) - \chi f_0(\gamma,0)$, respectively. Here $f_2(\alpha,\gamma,0)$ is obtained from $f_1(\alpha,\gamma,y)$ if $f_0(\gamma,y)$ is replaced by $R_0(\gamma,y)$, y set equal to zero and

$$R_0(\gamma,y) = [\gamma(\beta+\chi)\,\mathrm{ch}\,\gamma h - (\gamma^2+\beta\chi)\,\mathrm{sh}\,\gamma h]\,\mathrm{ch}\,\gamma y + \chi_1(\beta\,\mathrm{ch}\,\gamma h - \gamma\,\mathrm{sh}\,\gamma h)\,sh\,\gamma y$$

The rise of the free surface ζ and of the interface ζ_1 in the case of a two-layer fluid and the rise of the free surface ζ_* for a homogeneous fluid (i.e. when $\rho_1\to\rho$) are computed in the direction of the x-axis for the following values of the parameters: $\varepsilon = 0.2$ m, $H = 2.0$ m, $l = 6.0$ m, $h = 0.1$ m, $\rho = 1000.0$ kg/m^3, $a = 2$ m, $\omega = 4.34$ sec^{-4}, $k = 1.0$ m^{-1}, $\rho_1 = 1025.0$ kg/m^3, and given in the table below (all quantities are expressed in meters)

$x =$	-3	-1	1	3	5
$\zeta =$	0.0009	0.0053	0.0297	0.0210	0.0174
$\zeta_1 =$	0.0017	0.0108	0.0591	0.0418	0.0339
$\zeta_* =$	0.0008	0.0049	0.0275	0.0194	0.0161

Thus we see that the presence of inhomogeneity leads to an increase in the free surface of the fluid (in the present case by 8%), and the presence of a dock on the fluid surface reduces the amplitude of the surface and internal waves in the region between the plate and the oscillating section of the bottom. Moreover, the wave amplitudes on the interface are larger than the corresponding wave amplitudes on the free surface.

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HEAT TRANSFER INTO A SEMI-INFINITE REGION WITH A BOUNDARY MOVING ACCORDING TO AN ARBITRARY LAW

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We present an improvement to the previously given [1] method of solving certain problems with a moving boundary. We shall investigate an unsteady heat transfer in a semi-infinite region with a moving boundary at a specified temperature and zero initial conditions.

Using a coordinate system attached to the moving boundary, we describe the process in the form of the following problem:

$$\left[\frac{\partial}{\partial t}-\frac{\partial^2}{\partial x^2}-u(t)\frac{\partial}{\partial x}\right]T=0,\quad 0\leqslant x\leqslant\infty,\quad 0<t<\infty$$

$$T|_{x=0}=T_0(t),\quad T|_{x=\infty}=0,\quad T|_{t=0}=0$$

where u is the velocity of the moving boundary. The only quantity to be determined is the temperature gradient at the boundary of the region $q_0=(\partial T/dx)_{x=0}$.

In contrast to [1] we make the substitution

$$T=\theta\exp\int_0^t\frac{1}{4}u^2\,dt \tag{1}$$

We now obtain the following problem for θ :

$$\left[\frac{\partial}{\partial t}-\frac{\partial^2}{\partial x^2}-u(t)\frac{\partial}{\partial x}-\frac{u^2(t)}{4}\right]\theta=0,\qquad 0\leqslant x<\infty,\quad 0<t<\infty$$

$$\theta|_{x=0}=\theta_0=T_0\exp\int_0^t\frac{1}{4}u^2dt \tag{2}$$

Equation (2) can be written in the form [2]

$$\left(M-\frac{\partial}{\partial x}\right)\left(L+\frac{\partial}{\partial x}\right)\theta=0 \tag{3}$$